**MSDM5004 Part II Assignment 1**

**Due: 24 Apr 2022, 23:59**

1. Fourier filtering and smoothing

In Canvas, there is a file called HK\_Meteorological\_Data\_Daily\_1884-2021.csv, which contains daily meteorological data of Hong Kong from 1884 to 2021.

(a) Write a program that makes a graph of the daily mean temperature as a function of time. You should see that the mean temperature has fluctuated on a regular cycle for as long as observations have been recorded. What is the period of the cycle?

(b) Calculate the Fourier transform of the daily mean temperature and then make a graph of the magnitude squared of the Fourier coefficients as a function of (also called the power spectrum). You should see that there is a noticeable peak in the power spectrum at a zero frequency and at a nonzero value of . Explain why there is a peak at *k* = 0. The appearance of this peak at nonzero *k* tells us that there is one frequency in the Fourier series that has a higher amplitude than the others around it --- meaning that there is a large sine-wave term with this frequency, which corresponds to the periodic wave you can see in the original data. Find the approximate value of to which this peak corresponds. What is the period of the sine wave with this value of ? You should find that the period corresponds roughly to the length of the cycle that you estimated in part (a). This kind of Fourier analysis is a sensitive method for detecting periodicity in signals. Even in cases where it is not clear to the eye that there is a periodic component to a signal, it may still be possible to find one using a Fourier transform. In order to see the other smaller peaks more clearly, also make a plot showing the power spectrum in logarithmic scale (i.e., vs *k*).

(c) Now set all Fourier coefficients to zero except those of the lowest 10% of frequencies. Then calculate the inverse Fourier transform and plot it on the same graph as the original data. Comment on what you see. What is happening when you set the Fourier coefficients to zero?

(d) Modify your program so that it sets all Fourier coefficients to zero except those of the lowest 2% of frequencies, and run it again and compare the result with that of (c).

2. Consider the point-spread-function (PSF) defined for integer

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The PSF is the blurred image of the latent clear point image at :

Now consider the case that the undistorted clear image should have been

(a) Find the blurred image due to the PSF by linear convolution.

(b) Set . Find the circular convolution

for ..

(c) Draw (or plot) and .